A General Method Based on Harmonic Balance Techniques to Simulate Noise in Free Running Oscillators

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Abstract

A new approach using perturbation theory for simulating the noise behavior in free running microwave oscillators based on a piecewise harmonic balance technique is outlined and applied to a planar integrated microwave oscillator at 14 GHz. A single–sideband phase noise of -90 dBc/Hz at an offset frequency of 100 kHz was measured. Simulated and measured single–sideband phase noise agree within the accuracy of measurements. The method is neither limited to certain circuit topologies nor to certain nature of noise sources.

1. Introduction

The noise behavior is besides the signal properties essential for the design of microwave oscillators. While the determination of the signal behavior of oscillators is state of the art [1,2,3,4], this is not the case if the noise sources are taken into account.

We propose a new approach based on a piecewise harmonic balance technique to calculate the single–sideband phase noise of oscillators which is neither limited to a certain kind of topology of the circuit nor to certain nature of noise sources. In oscillators of technical interest the noise sources are small compared with the signals. Therefore the system of equations is linearized around the steady state. Due to the lack of phase reference in oscillators the resulting Jacobian is singular at the steady state and ill–conditioned for a small frequency deviation from the carrier frequency where we want to know the phase noise. We overcome this problem by using an eigenvalue decomposition of the Jacobian where the small eigenvalue due to the ill–condition of the matrix is taken into account. A simple equation for the simulation of the single–sideband phase noise $L(f_m)$ can be derived which allows to compute $L(f_m)$ in a numerically stable way.

2. Method of the Noise Analysis

As usual for the piecewise harmonic balance method the circuit is divided into a nonlinear and a linear subcircuit. The state variables in the nonlinear subcircuit and the voltages at the common ports, denoted with $\mathbf{U}$, are determined so that Kirchhoff's current law (KCL) is fullfilled.

$$\mathbf{F}(\mathbf{U}^0, \omega_0) = \mathbf{0} \tag{1}$$

Taking the noise sources into account, which are small compared with the signals, the voltages at the ports, the state variables of the nonlinear subcircuit and the frequency of oscillation vary only by a small value from the steady state,

$$\begin{aligned} \mathbf{U}_T(\omega) &= \mathbf{U}_T^0(\omega) + \delta\mathbf{U}_T(\omega); \quad && \omega = \omega_0 + \omega_m \\ \| \delta\mathbf{U}_T(\omega) \| &\ll \| \mathbf{U}_T^0(\omega) \|; && \omega_m \ll \omega_0. \end{aligned} \tag{2}$$

Thus the system of the nonlinear equations can be linearized around the steady state.

$$\mathbf{J}(\mathbf{U}_T^0, \omega)\delta\mathbf{U}_T + \mathbf{G}(\mathbf{U}_T^0)\mathbf{N}_T = \mathbf{0} \tag{3}$$

$$\text{with } \mathbf{J}(\mathbf{U}_T^0, \omega) \equiv \frac{\partial \mathbf{F}(\mathbf{U}_T, \omega)}{\partial \mathbf{U}_T}\Big|_{\mathbf{U}_T = \mathbf{U}_T^0} \tag{4}$$

The matrix $\mathbf{J}(\mathbf{U}_T^0, \omega)$ represents the Jacobian and the matrix $\mathbf{G}(\mathbf{U}_T^0)$ denotes the contributions of the noise sources $\mathbf{N}_T$ in each KCL equation. The index T denotes the time windowed signals as amplitude spectra of random signals may only be defined for time limited probes of the signals [5]. We use an ansatz where all Fourier coefficients and the frequency of oscillation are perturbed. Therefore all noise processes including the upconversion of $1/f^\alpha$ noise sources and the AM to PM conversion are taken into account to calculate the single–sideband phase noise.

In the vicinity of the carrier frequency, where the single–sideband phase noise is of interest, the linear system of equations is ill–conditioned. This results from a singular Jacobian at the steady state and due to the small frequency deviation f_m of the carrier frequency the deviations of the matrix elements are small and the condition number remains high [6]. In the vicinity of the carrier frequency means a frequency deviation of about 100 Hz to 10 MHz from the carrier frequency. Considering e.g. a 10 GHz oscillator this is a deviation of only 10^{-8} to 10^{-3} times the frequency of oscillation. To overcome this problem the system of equations is linearized with respect to the frequency and an eigenvalue decomposition of the Jacobian is used. Thus the complete correlation spectra can be calculated in a numerically stable way.

$$\mathbf{J}(\mathbf{U}_T^0, \omega) = \mathbf{J}(\mathbf{U}_T^0, \omega_0) + \omega_m \cdot \mathbf{J}_\omega(\mathbf{U}_T^0, \omega_0), \tag{5}$$

$$\text{with } \mathbf{J}_\omega(\mathbf{U}_T^0, \omega_0) \equiv \frac{\partial \mathbf{J}(\mathbf{U}_T^0, \omega)}{\partial \omega}\Big|_{\omega=\omega_0} \tag{6}$$

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The left– and right–sided eigenvectors of the unperturbed Jacobian $\mathbf{J}(\mathbf{U}_T^0, \omega_0)$ are denoted with $\mathbf{V}_j$ and $\mathbf{W}_i$ respectively.

$$\mathbf{V}_j^+ \cdot \mathbf{W}_i = \delta_{ij} \quad \text{with}: \ \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{7}$$

The eigenvalue of the unperturbed Jacobian which is zero is denoted with λ_1 and the corresponding eigenvectors with $\mathbf{V}_1$ and $\mathbf{W}_1$. It can be shown that the eigenvector $\mathbf{W}_1$ is determined by the steady state, $\mathbf{W}_1 = j\omega_0 \mathbf{K} \mathbf{U}_T^0$, where $\mathbf{K}$ is a diagonal matrix consisting of the number of each harmonic. Transforming $\mathbf{W}_1$ in the time domain shows that $\mathbf{w}_1(t)$ is the tangent vector to the steady state $\mathbf{u}^0(t)$. A two dimensional phase space with a limit cycle and a possible configuration of the eigenvectors at one time point is depicted in fig. 1.

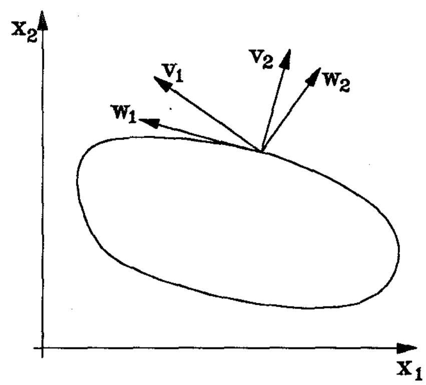

Figure 1: Two dimensional phase space with a limit cycle and eigenvectors $\mathbf{v}_i(t)$ and $\mathbf{w}_i(t)$.

The eigenvectors $\mathbf{W}_i$ are a base of the phase space and due to equation (7) a multiplication of $\mathbf{V}_1^+$ with a vector within the phase space is a projection onto the eigenvector $\mathbf{W}_1$ and therefore to the tangent vector of the steady. That means, the projection operator $\mathbf{W}_1\mathbf{V}_1^+$ applied to a vector named $\mathbf{z} = \sum_{i=1}^{N} a_i \mathbf{W}_i$ results in a vector tangential to the limit cycle with a length of the coefficient a_1. The perturbation of the eigenvalue $\lambda_1 = 0$, denoted with $\delta\lambda_1$, is expressed with terms up to the first order of the frequency deviation $\omega_m = 2\pi f_m$.

$$\delta\lambda_1 = 2\pi f_m \mathbf{V}_1^+ \mathbf{J}_\omega(\mathbf{U}_T^0, 2\pi f_0) j 2\pi f_0 \mathbf{K} \mathbf{U}_T^0 \tag{8}$$

The inverse of the Jacobian $\mathbf{J}(\mathbf{U}_T^0, 2\pi f_0)$ is represented by the eigenvalues and the left– and right–sided eigenvectors.

$$\mathbf{J}^{-1}(\mathbf{U}_T^0, 2\pi f_0) = \sum_{i=1}^{N} \frac{1}{\lambda_i} \mathbf{W}_i \mathbf{V}_i^+ \tag{9}$$

Using the inverse of the perturbed Jacobian with the perturbed eigenvalues and eigenvectors the voltage fluctuations and thus the correlation spectra can be calculated. Taking only the term with the major contribution to the correlation spectra into account, that is the term with the small eigenvalue $\delta\lambda_1$ and the corresponding eigenvectors $\mathbf{V}_1$ and $\mathbf{W}_1$, we derive a simple equation for the single–sideband phase noise $L(f_m)$.

$$L(f_m) = \frac{1}{(2\pi f_m)^2} \cdot \frac{\mathbf{V}_1^+ \mathbf{C}^{GN}(f_0 + f_m) \mathbf{V}_1}{|\mathbf{V}_1^+ \mathbf{J}_\omega(\mathbf{U}^0, 2\pi f_0) \mathbf{K} \mathbf{U}^0|^2} \tag{10}$$

The derivation of this equation is demonstrated in detail in [7,8] based on a nodal oriented harmonic balance technique. The correlation spectra of the noise sources multiplied with the matrix $\mathbf{G}(\mathbf{U}_T^0)$ are denoted by the matrix $\mathbf{C}^{GN}(f_0 + f_m) = \mathbf{G}(\mathbf{U}_T^0)\mathbf{C}^N(f_0 + f_m)\mathbf{G}^+(\mathbf{U}_T^0)$. $\mathbf{V}_1$ is the solution of a homogeneous linear system of equations, $\mathbf{J}^+(\mathbf{U}_T^0, 2\pi f_0)\mathbf{V}_1 = \mathbf{0}$, which can be obtained very easily with a standard LU-decomposition of the Jacobian. The derivative of the Jacobian with respect to the frequency $\mathbf{J}_\omega(\mathbf{U}^0, 2\pi f_0)$ can be calculated numerically as we will show in our example. The denominator of the second term is constant and needs to be calculated only once. The numerator consists of the correlation spectrum of the noise sources multiplied with the vector $\mathbf{V}_1^+$ from the left side and with $\mathbf{V}_1$ from the right side. As we already described, this multiplication is a projection of all the noise sources of the phase space onto the tangent vector to the steady state. That means the vector $\mathbf{V}_1$ filters the contributions of the noise sources which are tangential to the steady state and therefore induce the phase noise.

This method results in a complete calculation of the phase noise of free runing oscillators, where all effects of the harmonic signals mixed with the noise sources and the non stationarity of noise sources are taken into account [9].

3. Simulation and Measurement of the Single–Sideband Phase Noise of a Microwave Oscillator at 14 GHz

This new method is applied to a planar [10] integrated microwave oscillator at 14 GHz with a GaAs MESFET. The equivalent circuit of the MESFET (fig. 2) was obtained by S–parameter measurements at several bias points.

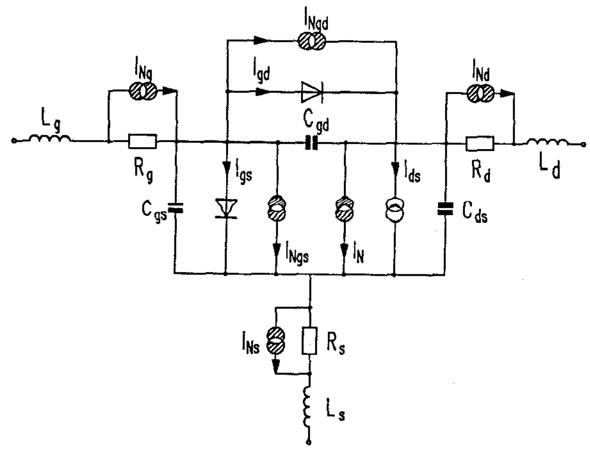

Figure 2: Equivalent circuit of the GaAs MESFET NE710.

A modified SPICE model [11,12] was used to characterize the nonlinearities of the used MESFET. The white noise sources are thermal noise sources of the losses or shot noise sources of the internal diodes of the transistor. The NF–noise power was measured for several bias voltages and a $1/f^{\alpha}$–noise source was modelled. The measured NF–noise power is depicted in fig. 3 for a voltage of -0.7 V between gate and source and 3.0 V between drain and source.

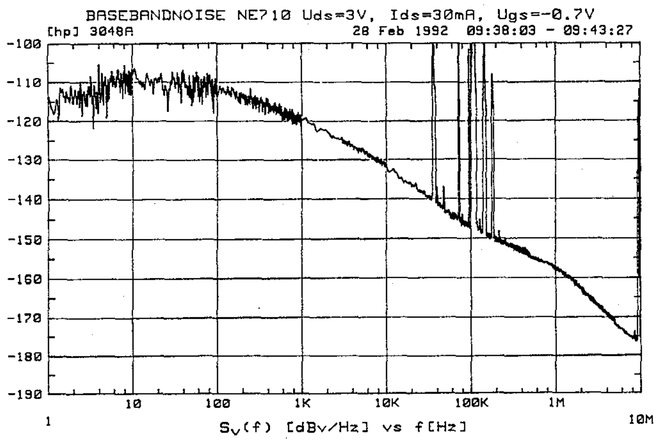

Figure 3: NF-noise measurement with $U_{GS} = -0.7$ V and $U_{DS} = 3.0$ V.

The correlation spectrum of the $1/f^{\alpha}$–noise source is given by

$$\mathbf{C}^{f} = \frac{c(U_{GS}, U_{DS}) \cdot (10\mathrm{kHz})^{\alpha}}{|f_m|^{\alpha}}. \tag{11}$$

The function $c(U_{GS}, U_{DS})$ denotes the spectral noise power at a frequency of 10 kHz in dependence of the gate–source and the drain–source voltage. The exponent α was obtained by averaging the slope of the measured baseband noise between 1kHz and 100 kHz over several bias points.

The linear circuit was designed with microstrip lines for a frequency of oscillation at 14 GHz. The designed circuit is depicted in fig. 4.

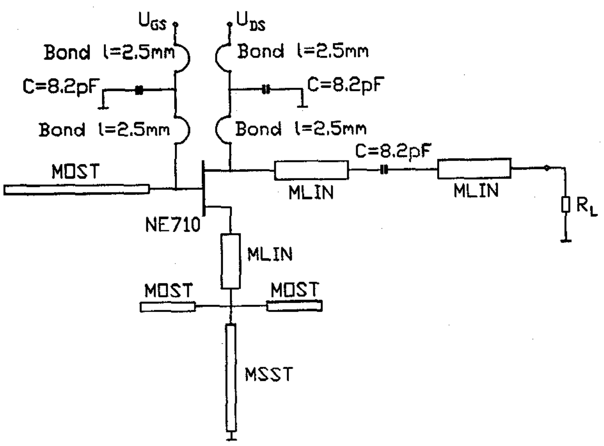

Figure 4: The oscillator circuit.

A photography of the oscillator is shown in fig. 5.

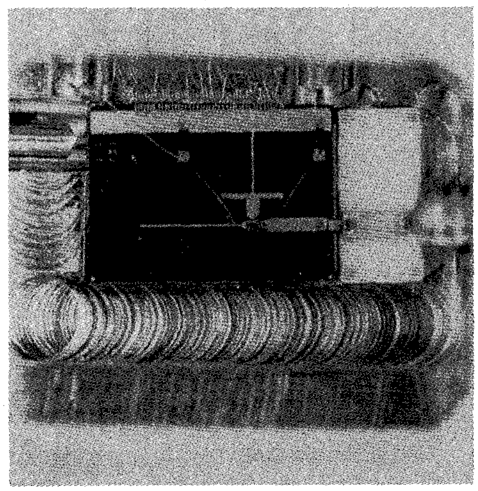

Figure 5: Photography of the oscillator.

The spectrum of the output power measured with the spectrum analyzer HP71000 is shown in fig. 6 with a maximum power of 12.85 dBm at 14.2 GHz. A 10 dB attenuator was used at the input port of the spectrum analyzer.

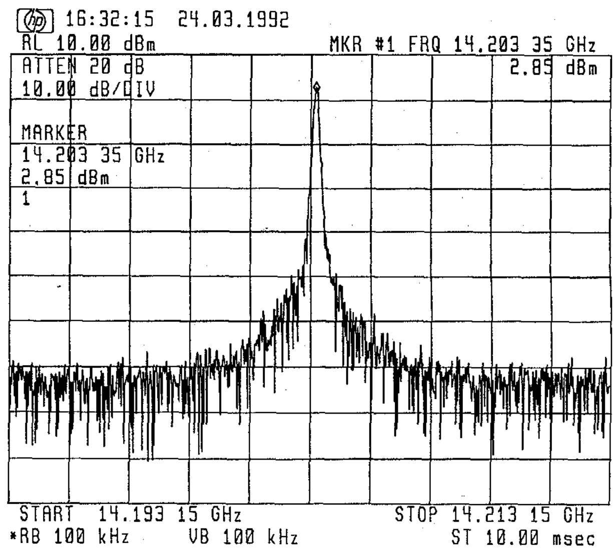

Figure 6: The spectrum of the oscillator.

The equivalent noise sources at the ports were simulated with the linear network analysis program SANA [13]. Hence the correlation matrices of all noise sources are known. By solving Kirchhoffs current law in order to obtain the system of equations the matrix $\mathbf{G}(\mathbf{U}_T^0)$ is automatically obtained if the noise sources are taken into account in the equivalent circuit. The vector $\mathbf{V}_1$ is calculated by solving the linear system of equations $\mathbf{J}^{+}\mathbf{V}_1 = \mathbf{0}$ with a standard LU–decomposition.

As the numerical differentiation of the Jacobian with respect to the frequency is not sensitiv to the choice of the frequency shift a simple numerical differentiation algorithm can be used. The noise power of the oscillator was measured with the noise measurement system HP3048 from Hewlett Packard by using the frequency discriminator method. We obtain a single-sideband phase noise $L(f_m)$ of -90 dBc/Hz at $f_m = 100$ kHz. The simulated and measured single-sideband phase noise is depicted in fig. 7 where only one harmonic was taken into account to simulate $L(f_m)$.

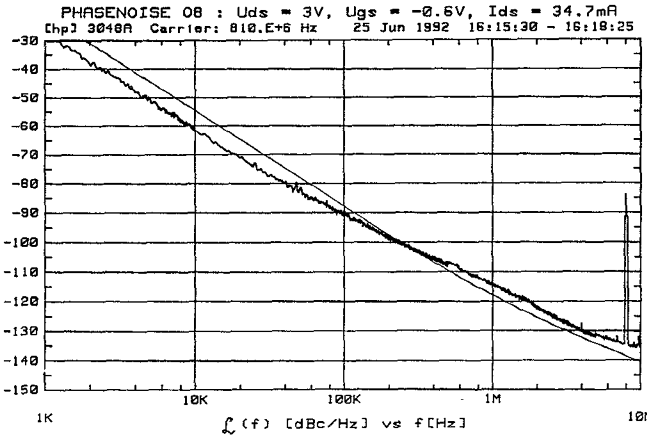

Figure 7: Measured and simulated single-sideband phase noise $L(f_m)$.

At small frequency deviations the single-sideband phase noise $L(f_m)$ decreases with 33 dB per decade due to the modelled factor $\alpha = 1.3$ of the $1/f^{\alpha}$ noise source. $L(f_m)$ decreases with 20 dB per decade due to the white noise sources for a frequency deviation greater than 1 MHz. The deviation of the simulated and the measured single-sideband phase noise is under 5 dB over the whole measured frequency range from 1 kHz to 10 MHz. Another important feature of our method is the low numerical effort to calculate the noise behavior of oscillators. A HP9000 workstation needs just about 6 seconds to calculate 50 points of the single-sideband phase noise without any optimization done to minimize the computation time.

3. Conclusion

We demonstrated a new approach based on a piecewise harmonic balance technique to simulate the single-sideband phase noise in free running microwave oscillators. The method was applied to a planar integrated microwave oscillator at 14 GHz. A single-sideband phase noise of -90 dBc/Hz at an offset frequency of 100 kHz was obtained by using only microstrip lines at the gate and source as resonators. The difference of the simulated and measured single-sideband phase noise lies within the accuracy of measurements over the whole measured frequency range between 1 kHz and 10 MHz. The method proved to be a fast, reliable and numerically stable tool for the design of microwave oscillators.

Acknoledgement

The autors would like to thank Dipl.-Ing. J. Schaffer and Dipl.-Ing. V. Güngerich who designed and built up the oscillator.

This work was supported by the Deutsche Forschungsgemeinschaft.

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